



## Department of Mathematics and Statistics

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# Math 117 - FALL 2024 - Common Final Exam

Print name: \_\_\_\_\_

Section number: \_\_\_\_\_ Instructor's name: \_\_\_\_\_

### Directions:

- This exam has 14 questions. Please check that your exam is complete, but otherwise keep this page closed until the start of the exam is called.
- Fill in your name, and your instructor's name.
- It will be graded out of 100 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.
- You may use a calculator which does not allow internet access. The use of any notes or electronic devices other than a calculator is prohibited.
- **Unless otherwise stated, round any constants to three decimal places if necessary.**

**Good luck!**

|           |   |    |    |    |    |    |   |       |
|-----------|---|----|----|----|----|----|---|-------|
| Question: | 1 | 2  | 3  | 4  | 5  | 6  | 7 | 8     |
| Points:   | 5 | 6  | 4  | 10 | 12 | 4  | 8 | 9     |
| Score:    |   |    |    |    |    |    |   |       |
| Question: | 9 | 10 | 11 | 12 | 13 | 14 |   | Total |
| Points:   | 6 | 6  | 9  | 6  | 6  | 9  |   | 100   |
| Score:    |   |    |    |    |    |    |   |       |

1. (5 points) Let  $f$  be an increasing function defined for  $x \geq 0$ . The table gives values of  $f(x)$  at selected values of  $x$ .

|        |    |   |    |    |    |
|--------|----|---|----|----|----|
| $x$    | 1  | 2 | 3  | 4  | 5  |
| $f(x)$ | -4 | 6 | 10 | 23 | 40 |

The function  $g$  is given by

$$g(x) = \frac{x^3 - 7}{x + 2}.$$

- (a) The function  $h$  is defined by  $h(x) = g(f(x))$ . Find the value of  $h(2)$  as a decimal approximation, or indicate that it is not defined.

$$h(2) = g(\underbrace{f(2)}_6) = g(6) = \frac{6^3 - 7}{6 + 2} = \frac{209}{8} = 26.125$$

(3 points)

- (b) Find the value of  $f^{-1}(-4)$ , or indicate that it is not defined.

$$f^{-1}(-4) = 1$$

(2 points)

2. (6 points) Find a possible formula for the polynomial function shown below.

4th degree (1 point)

$$y = a \cdot (x+5)^2(x+1)(x-8)$$

(2 points)

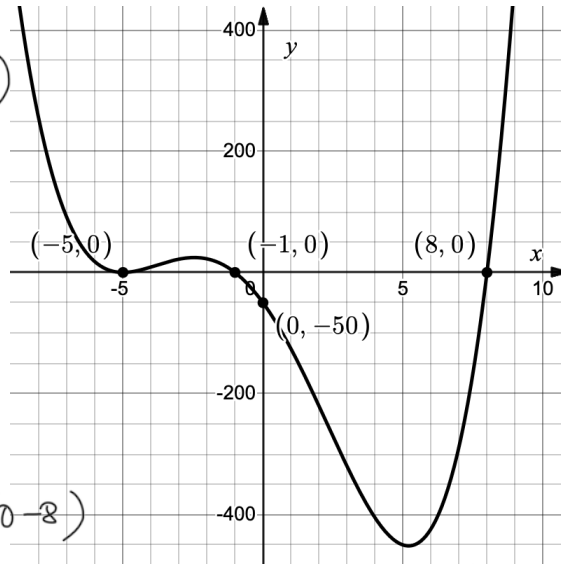
Using  $(0, -50)$   
to find  $a$

(2 points)

$$-50 = a \cdot (0+5)^2 \cdot (0+1) \cdot (0-8)$$

$$-50 = a \cdot (5)^2 \cdot (1) \cdot (-8)$$

$$a = \frac{1}{4}$$

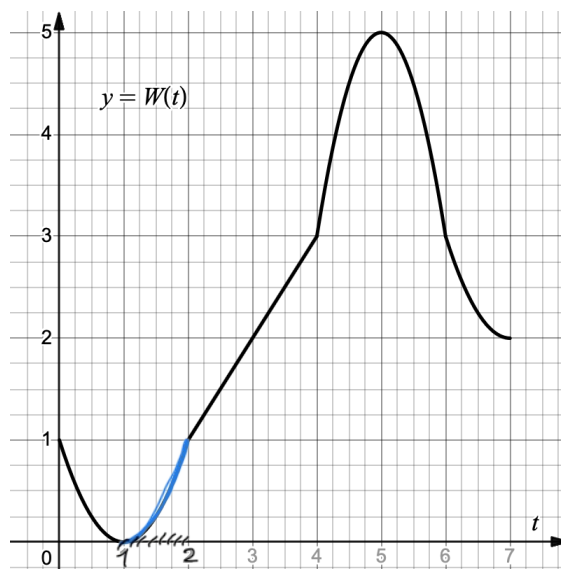


final formula

$$y = \frac{1}{4} \cdot (x+5)^2 \cdot (x+1) \cdot (x-8)$$

(1 point)

3. (4 points) The graph of  $y = W(t)$  is shown for  $0 \leq t \leq 7$ . What are all intervals of  $t$  on which the function is both increasing and concave up? If none, write DNE.



$$1 \leq t \leq 2$$

4. (10 points) Let  $k(x)$  be the function given by

$$k(x) = \frac{x^2 - 2x - 35}{x^3 - 3x^2 - 18x} = \frac{(x+5) \cdot (x-7)}{x \cdot (x+3) \cdot (x-6)}$$

- (a) Find the zeros of  $k(x)$ .  $x^2 - 2x - 35 = 0$   
(2 points)  $(x+5)(x-7) = 0$  (or quadratic formula)  
 $x = -5, x = 7$

- (b) Find all vertical asymptotes.  
(3 points)  $x^3 - 3x^2 - 18x = 0$   
 $x(x+3)(x-6) = 0$   
 $x = 0, x = -3, x = 6$

- (c) What is the domain of  $k(x)$ ?  
(2 points)  $x \neq 0, x \neq -3, x \neq 6$

- (d) Find a horizontal asymptote and describe the long-run behavior of the function  $k(x)$  as  $x \rightarrow +\infty$  and  $x \rightarrow -\infty$ .

(3 points)  $k(x) \approx \frac{x^2}{x^3} = \frac{1}{x} \rightarrow 0$   
as  $x \rightarrow \pm \infty$   
horizontal asymptote  $y = 0$

5. (12 points) A theater manager graphed weekly profits as a function of the number of patrons and found that the relationship was linear. One week the profit was \$9500 when 1300 patrons attended. Another week 1500 patrons produced a profit of \$11,400.

- (3) (a) Find a formula for weekly profit,  $y$ , as a function of the number of patrons,  $x$ .

$$y = m \cdot x + b, \quad m = \frac{11400 - 9500}{1500 - 1300} = \frac{1900}{200} = 9.5$$

$$b = 9500 - 9.5 \cdot 1300 = -2850$$

$$y = 9.5 \cdot x - 2850$$

- (3) (b) Interpret the slope and the  $y$ -intercept.

$$m = 9.5 \frac{\$}{\text{patron}}$$

$$b = -2850 \text{ negative profit when zero patrons}$$

- (3) (c) If the weekly profit was \$20,900, how many patrons attended the theater?

$$20900 = 9.5 \cdot x - 2850 \Rightarrow 20900 + 2850 = 9.5x$$

$$23750 = 9.5 \cdot x \Rightarrow \frac{23750}{9.5} = x = 2500 \text{ patrons}$$

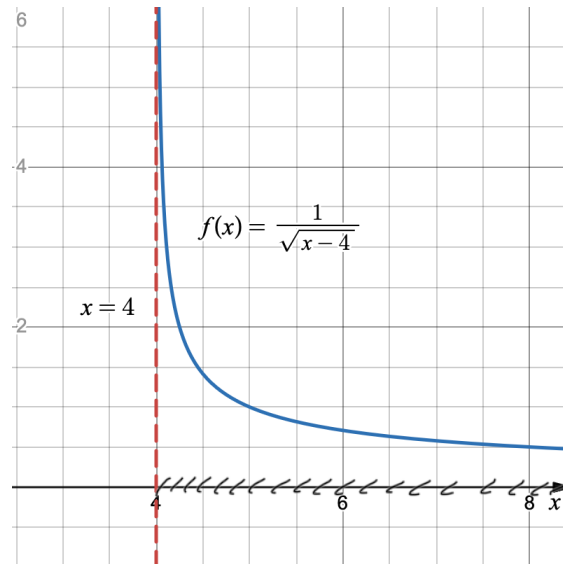
- (3) (d) Find a formula for the number of patrons as a function of profit.

$$x = \frac{y + 2850}{9.5}$$

6. (4 points) Find the domain of the function

$$f(x) = \frac{1}{\sqrt{x-4}}$$

by examining its formula and/or its graph.



$$x-4 > 0$$

$$x > 4$$

7. (8 points) Let

$$f(x) = \frac{5x-2}{1-3x}$$

For what value of  $x$  is  $f(x) = 2$ ?

$$(2) \quad \frac{5x-2}{1-3x} = 2$$

$$(4) \quad \begin{cases} 5x-2 = 2 \cdot (1-3x) \\ 5x-2 = 2-6x \\ 5x+6x = 2+2 \end{cases}$$

$$(2) \quad \begin{aligned} 11x &= 4 \\ x &= \frac{4}{11} \end{aligned}$$

8. (9 points) A water fountain shoots a stream of water from ground level into the air forming a parabolic arc. The height  $H(t)$ , in feet, of the water stream at time  $t$ , in seconds, is modeled by the equation

$$H(t) = -16t^2 + 48t = a \cdot (x-h)^2 + k$$

$$a = -16, b = 48, c = 0$$

- (3) (a) Find the time it takes for the water to reach its maximum height.

$$h = -\frac{b}{2a} = -\frac{48}{2 \cdot (-16)} = \left(\frac{3}{2}\right)$$

- (3) (b) What is the maximum height of the water?

$$k = H\left(\frac{3}{2}\right) = -16 \cdot \left(\frac{3}{2}\right)^2 + 48 \cdot \left(\frac{3}{2}\right) = -36 + 72 = 36$$

- (3) (c) Find the time it takes for the water stream to come back to ground level.

$$-16t^2 + 48t = 0$$

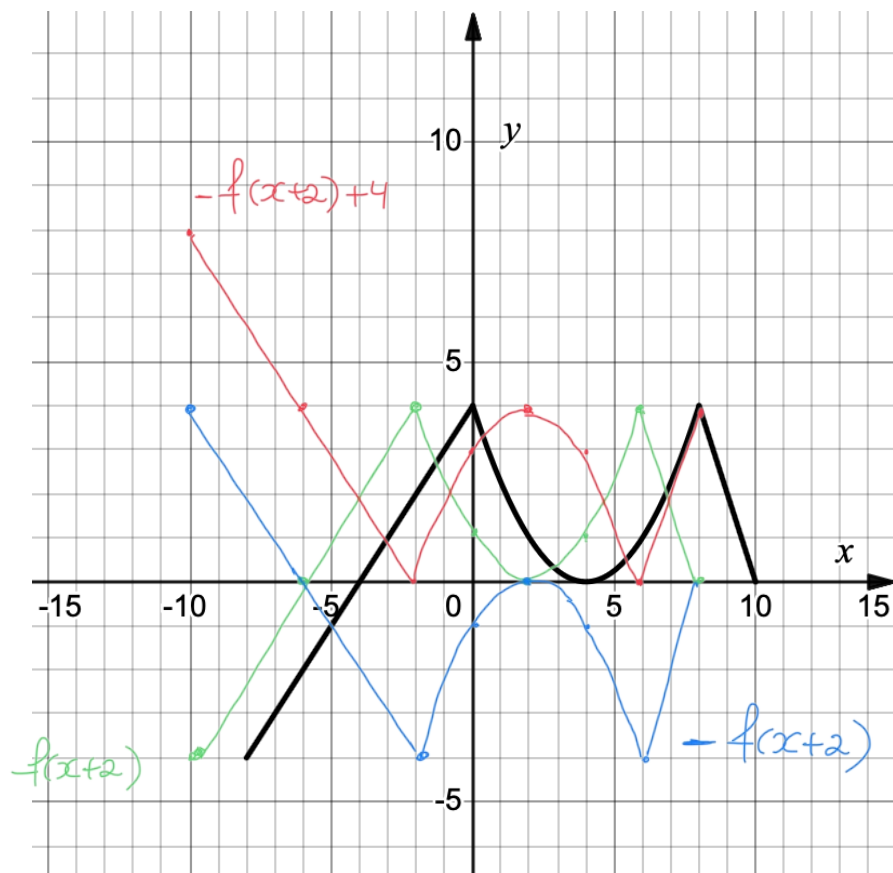
$$-16t \cdot (t-3) = 0$$

$$t = 0, \quad t = 3$$

9. (6 points) The graph of the function  $f(x)$  is given below. Graph the following transformation of the function  $f(x)$  on the same axes.

$$y = -f(x+2) + 4$$

2 points  
for each  
correct  
transformation



10. (6 points) The point  $(2, -3)$  is on the graph of  $y = p(x)$ . Give the coordinates of the corresponding point on the graph of the transformation

$$q(x) = -3 \cdot p(2x) + 1.$$

$$p(x): (2, -3)$$

$$p(2x): (1, -3)$$

$$-3 \cdot p(2x): (1, 9)$$

$$-3 \cdot p(2x) + 1: (1, 10)$$

2 points for each  
correct transformation

11. (9 points) At the Fun Slice Pizzeria it costs 2 cents per square inch to add pepperoni to a pizza.

(a) Find a formula for the cost  $C(A)$  to cover  $A$  square inches of a pizza.

$$(3) \quad C(A) = A \cdot 0.02$$

(b) The area of a circular pizza (in square inches) is a function of the radius  $r$  (in inches) is given by

$$A(r) = \pi r^2.$$

Find an expression for  $C(A(r))$  and explain what it means in practical terms.

$$(3) \quad C(A(r)) = \pi \cdot r^2 \cdot 0.02$$

(c) Find the cost in dollars of completely covering a circular pizza of radius 8 inches. Round your answer to the nearest penny.

$$(3) \quad C(A(8)) = \pi \cdot 8^2 \cdot 0.02 = \$4.02$$

12. (6 points) In a microwave oven, cooking time  $t$  is inversely proportional to the square root of the amount of power used  $w$ . It takes 8 minutes to heat a frozen dinner at 625 watts.

$$t = \frac{k}{\sqrt{w}}$$

- (3) (a) Write a formula for the cooking time,  $t$ , as a function of power level,  $w$ .

$$8 = \frac{k}{\sqrt{625}} \Rightarrow 8 \cdot 25 = k \Rightarrow t = \frac{200}{\sqrt{w}}$$

$200 = k$

- (3) (b) Find the cooking time needed to heat the frozen dinner at the power level  $w = 324$  watts.

$$t = \frac{200}{\sqrt{324}} = \frac{200}{18} = 11.11 \text{ minutes}$$

13. (6 points) The function  $f$  is a rational function. Its graph is shown below. Give a possible formula for  $f(x)$ .

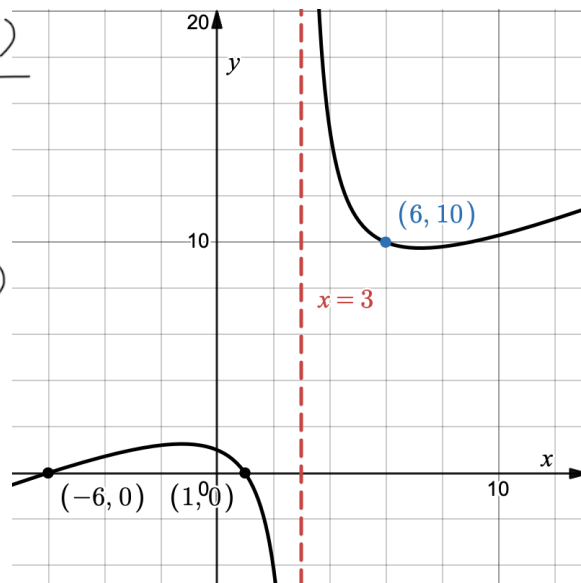
(2 points)

$$f(x) = a \frac{(x+6) \cdot (x-1)}{x-3}$$

(2 points)

Using the point  $(6, 10)$

$$10 = a \frac{(6+6) \cdot (6-1)}{(6-3)}$$



(1 point)

(1 point) final formula

Calculations

$$10 = a \frac{12 \cdot 5}{3} \Rightarrow$$

$$a = \frac{1}{2}$$

$$f(x) = \frac{1}{2} \frac{(x+6) \cdot (x-1)}{x-3}$$

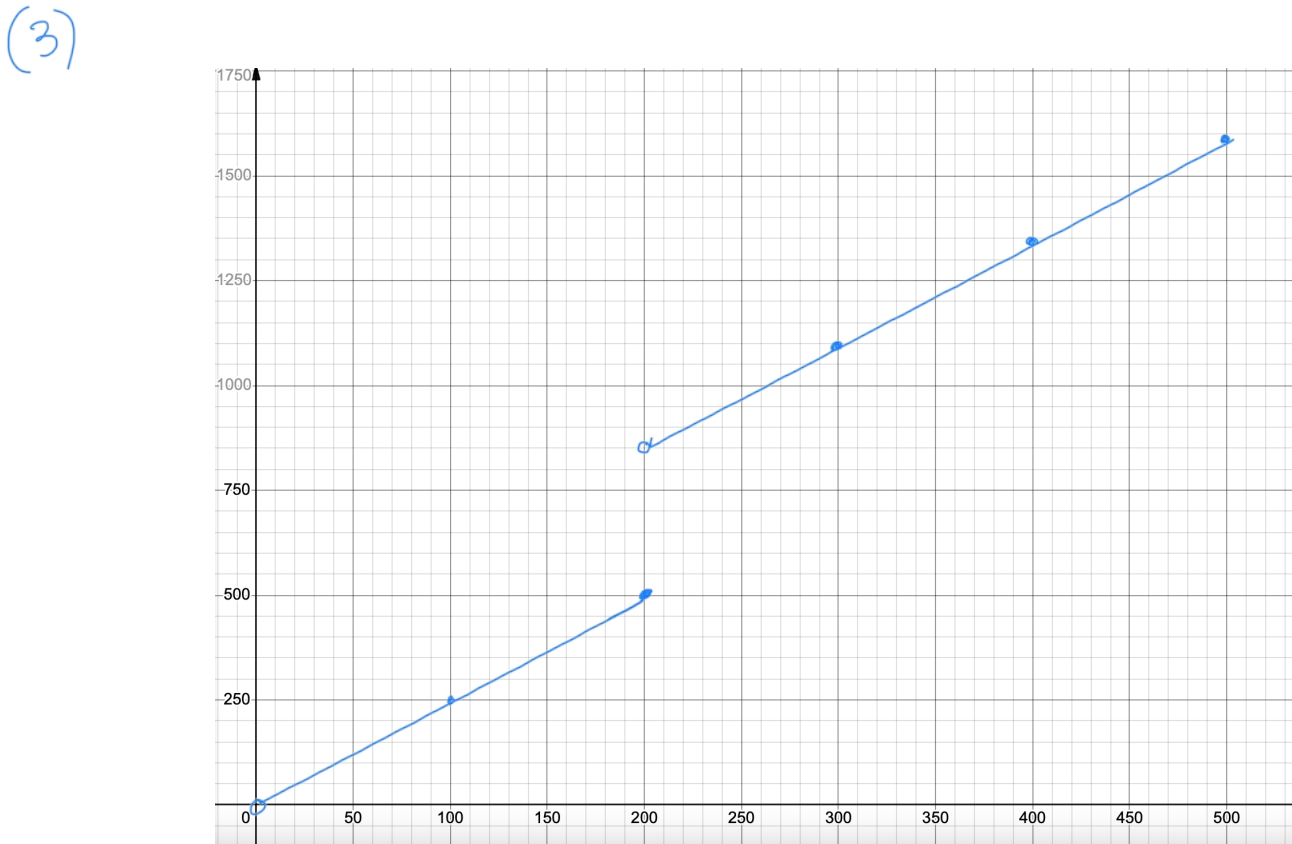
14. (9 points) A floor-refinishing company charges \$2.5 per square foot to strip and refinish a tile floor for up to 500 square feet. Since the tiles generate toxic waste, there is an additional charge of \$350 for toxic waste disposal for any job that includes more than 200 square feet of tile.

(a) Express the cost,  $y$ , of refinishing a floor as a piecewise-defined function of the number of square feet,  $x$ , to be refinished.

(3)

$$y = \begin{cases} 2.5 \cdot x & , \quad 0 < x \leq 200 \\ 2.5x + 350 & , \quad 200 < x \leq 500 \end{cases}$$

(b) Graph the function.



(3) (c) Give the domain and range.

Domain:  $0 < x \leq 500$

Range:  $0 < y \leq 500,$   
 $850 < y \leq 1600$

## Formulas

**Average rate of change:**  $\frac{f(b) - f(a)}{b - a}$

**Slope-intercept form:**  $y = b + mx$

**Point-slope form:**  $y - y_0 = m(x - x_0)$

**Standard form:**  $Ax + By = C$

**Quadratic function:**  $y = ax^2 + bx + c$

**Factored form:**  $y = a(x - r)(x - s)$

**Quadratic formula:**  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**Vertex form:**  $y = a(x - h)^2 + k$

**Power function**  $y = kx^p$

**Directly proportional:**  $y = kx$

**Inversely proportional:**  $y = \frac{k}{x}$

**Factored form of a polynomial:**  $p(x) = c(x - a_1)(x - a_2) \cdots (x - a_n)$